Course Outlines of Courses of MS/M. Phil. Mathematics

Course Title: Advanced Group Theory

Course Code: MATH 5101

Credit Hours: 3

Prerequisites: Algebra I

Course Contents:

- Some basic part: Dihedral groups, Symmetric groups, Quaternion group, Homomorphism and Isomorphism, Centralizer, Center and Normalizer, Cyclic groups, Quotient groups and homomorphisms, isomorphism theorems, etc.
- Group Actions, kernel, transitive group actions, group action by conjugation and class equation, Some results on finite p-groups,
- Automorphisms, The Sylow theorems, Simple groups,
- Direct product, Fundamental theorem of finitely generated Abelian groups, Table of groups of small order,
- Semi direct product, short survey about free groups
- P-groups,
- Nilpotent and solvable groups,
- Some example and theorem on basic representation theory.

Recommended Texts:


Course Title: Rings and Modules

Course Code: MATH 5102

Credit Hours: 3

Prerequisite: Algebra, Linear Algebra

Course Contents:

- **Rings**: Rings, Integral Domains, Subrings, Centre of a Ring, Characteristic, Ideals (Right, Left, Two-Sided), Examples, Ideal generated by a Set, Principal Ideal Ring, Sum and Product of Ideals, Left and Right Quotient, Regular Ring, Homeomorphisms, Kernel and Image, Correspondence Theorem, Embedding, Homeomorphisms Extensions, External and Internal Direct Sums, Direct Summand, Homeomorphisms Theorems, Quotient, Factorization of Homeomorphisms, Commutative Diagram, Nilpotent (Nil) Rings and Ideals, Sum and Quotient of Nilpotent (Nil)
Ideals, Maximal, Prime and Primary Ideals, Zorn’s Lemma and Applications, Boolean Ring, Nil Radical, Semi Prime and Minimal Prime Ideals.

- **Modules**: Modules, Direct Sum, Homeomorphisms Module generated by a set, FG Module, Basis, Free Module, Exact and Short Exact Sequence, Projected Module, Noetherian, Hilbert Module, Basis Theorem.

**Recommended Texts:**

1. D. M. Burton, Rings and Ideals.
2. T. W. Hungerford, Algebra.
3. T. S. Blyth, Module Theory.
4. I. N. Herstein, Topics in Algebra.

**Course Title: Commutative Algebra**

**Course Code:** MATH 5103

**Credit Hours:** 3

**Prerequisite:** Algebra

**Course Contents:**

**Modules**: Sub-modules and quotient modules, Operations on sub-modules, Direct Sum and direct product, Finitely generated modules, Module homomorphism, Isomorphism theorems, One-one Correspondence Theorem, Injective and projective modules, Baer’s Criterion, Tensor product of modules, Hom(-,-) of Modules, Modules of fractions, Modules over PID, Fundamental Theorem of finitely generated modules.

**Homologies**: Category and functors, Covariant and contra-variant functors, Left and right exact functors, Additive functors, Chain and co-chain complexes, syzygies, Homologies and co-homologies, Tensor product of complexes, Homological characterization of projective and injective Modules, Homotopy of Complexes, Long exact sequences of homologies, Snake and five Lemmas, Projective and Injective Resolutions, Ext and Tor modules, Properties of Ext and Tor Modules.

**Recommended Texts:**

2. Thomas W. Hungerford: Algebra.
5. Vermani, An elementary approach to homological algebra.

**Course Title: Combinatorial Commutative Algebra**
Course Code: MATH 5105

Credit Hours: 3

Prerequisite: Rings and Modules

Course Contents:

- **Review of Commutative Rings:**
- **Monomial Ideals:** Operations on ideals
- **Module Theory:** Graded modules, Operations on submodules, Direct Sum and direct product, Finitely generated modules, Algebras,
- **Rings of fractions:**
- **Noetherian rings and modules:**
- **Primary decomposition and associated prime ideals,**
- **Integral Closure of monomial ideals:**
- **Simplicial Complex**
- **Grobner bases:** Dickson’s Lemma, Hilbert basis theorem, Division algorithm, Buchberger’s criterion,
- **An introductory lecture on Singular**
- **Graded rings and modules:** Graded free resolution
- **Dimension Theory:** Hilbert function and Hilbert Series, The Koszul Complex, Depth, Cohen-Macaulay modules

Recommended Texts:

3. M. Gruel, G. Pfister, A Singular introduction to commutative algebra,
7. R. H. Villaraeal, Monomial Algebras.

Course Title: Advanced Functional Analysis

Course Code: MATH 5106

Credit Hours: 3

Prerequisite: Functional Analysis

Course Contents:

Recommended Texts:


Course Title: Advanced Numerical Analysis

Course Code: MATH 5201

Credit Hours: 3

Prerequisites: Numerical Analysis

Course Contents:

- Qusai-Newton method, Broyden method,
- **Solutions of Linear Systems**: Steepest decent method, Conjugate gradient method,
- **Approximation Theory**: Discrete least square approximation, Continuous least square approximation, Chebyshev Polynomials, Orthogonal Polynomials,
- **Numerical Integration**: Adaptive Quadrature, Gaussian Quadrature,

Recommended Texts:


Course Title: Advanced Differential Equations

Course Code: MATH 5202

Credit Hours: 3

Prerequisites: Calculus

Course Contents:

- Some basic part: Introduction to ODES, Classification of Differential Equations, Solutions and types of
solution, Slope fields and Integral curves,
- First order ODES and Solution, 2nd order ODES, Linear homogeneous and non-homogeneous ODES, Method of U.C, Variation of parameters, Reduction of order, Wronskian and fundamental set of solutions, Mathematical modelling.
- Series solution to ODES.
- Laplace Transform, basic theorems, convolution, solution of ODES by Laplace Transform, System of differential equations and solution.
- Fourier Series and fourier transform and solution of ODES.
- Solution of system of ODES, Matrix and Eigen values and Eigen vectors, phase potrait, Higher Order ODES.
- Some introduction to basic terminology of PDES, solution of PDES, Separation of variables
- Fourier series and PDES , Heat equation, Laplace equation, wave equation
- Advance Methods in PDES.

Recommended Texts:


Course Title: Iterative methods for solving nonlinear equations

Course Code: MATH 6205

Credit Hours: 3

Prerequisite: Numerical Analysis

Course Contents:

- Introduction, Classification of iteration functions, Order of convergence and related concepts, General theorems of iteration functions, Linear and superlinear convergences
- The fixed point method, its generalizations and relation with Kirik; Mann; Ishikawa; iterative methods
- Variants and generalizations of Newton-Raphson method, The formulas of Halley, House holder and Chebyshev, Predictor-Corrector methods
- Quadrature based iterative formulas
- Continued fractions based iterative formulas
- Functional equations, Adomian; Homotopy based iterative methods
- Solution of nonlinear system of equations, Fixed points for functions of several variables, Newton's method, Quasi-Newton methods, Steepest-Descent techniques, Homotopy and Continuation methods

Recommended Texts:


Course Title: Integral Inequalities

Course Code: MATH 5209

Credit Hours: 3

Prerequisite: Numerical Analysis

Course Contents:

- Numerical integration, Composite and Romberg numerical integration, Adaptive quadrature methods, Error analysis of quadrature rules
- Midpoint, Trapezoidal and Simpson inequalities for Lipschitzian mappings
- Montgomery identity, Ostrowski inequality for functions whose first derivatives are in $L_p$ and $L_1$ spaces and its applications
- Generalizations of Ostrowski inequalities and applications
- Chebyshev functional, Gruss and Gruss type inequalities and applications
- Gaussian and weighted Gaussian quadrature rules, Weighted Ostrowski and Gruss type inequalities

Recommended Texts: